An Integrated Constitutive Theory for the Mechanical Behaviour of Sea Ice: Micromechanical Interpretation

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INTRODUCTION

Ice in general, and columnar sea ice in particular, is a very complex material which exhibits a wide range of behaviors often at the same time. As a consequence of its occurrence at high homologous temperatures, the mechanical behavior of ice is strongly influenced by temperature and rate of loading. In most applications ice behaves as a material undergoing both continuum deformation and cracking activity. Microcracking activity can be effectively idealized as continuum behavior and represented by models describing multiaxial flow with appropriate modifications to account for the damage due to micro-cracking. This type of damage manifests itself as strain-softening observed during constant strainrate tests in compression and as tertiary creep in compressive creep tests. On the other hand, macrocracking activity implies "failure" of the material at macroscale and is a separate behavioral mechanism. A constitutive model that captures both the mechanisms of multiaxial flow as a continuum and ultimate failure by macrocracking in addition to the rate and temperature dependence is necessary to investigate the mechanics of deformation and progressive failure in ice.

Previous modeling work directly relevant to the development of such a constitutive theory is limited since most investigators have treated ice as either an elastic-plastic or a viscoelastic material under multiaxial states of stress. They include the works of Glen (1955), Palmer (1967), Shapiro (1978), Sinha (1978, 1979, 1983b), Michel (1981), Wang (1982), Morland and Spring (1981, 1982, 1983), and Ashby and Duval (1985). Studies on the damage behavior of ice have been relatively few in comparison. Sinha (1982) and Sanderson and Child (1984) have studied the occurrence of first cracks in ice under compressive creep conditions, while Schulson (1979, 1984a,b)

and Currier and Schulson (1982) have studied the brittle to ductile transition in polycrystalline ice. The use of damage mechanics theory for modeling ice is being explored by Karr (1984, 1985a,b). More recently, Szyszkowski and Glockner (1985) have proposed a relatively comprehensive model that describes the continuum behavior of pure ice accounting for damage due to microcracking. The models cited here are highly variable in the degree to which they are related to the micromechanics of ice behavior, i.e., the physical mechanisms governing ice behavior at microscale.

This paper presents a new rate-sensitive constitutive theory for describing the mechanical behavior of ice developed at MIT. The theory integrates (i) a potential function description for modeling deformations based on a nonlinear generalization of the Maxwell differential model and the associated flow rule with (ii) a rate-sensitive Drucker-Prager surface to describe ultimate failure by macrocracking representing either yielding of the material or fracture depending on the stress state. The constitutive model is characterized by its ability to:

(a) Decompose the various recoverable (instantaneous and delayed elastic or "primary creep") and irrecoverable (secondary creep and strain-softening or tertiary creep) components of strain.

(b) Represent continuously damaging or strain-softening material behavior during ductile-to-brittle transition in compression with a linear incremental damage accumulation model.

(c) Describe materially anisotropic mechanical behavior with a pressure-insensitive but rate-dependent potential function.

(d) Predict first crack occurrence or nucleation with a rate-dependent limiting tensile strain criterion.

(e) Distinguish the mechanisms of multiaxial flow as a continuum and ultimate failure by macrocracking leading to yielding of the material or fracture.

Further, the model shows strong dependency of the continuum behavior under creep and constant strainrate conditions. The model predictions compare very well with several independent sets of experimental data, particularly those for first-year sea ice. Data for the uniaxial "strength" of sea ice has been augmented with the extensive experimental data base available for pure crystalline ice through a normalization proposed by Weeks and Assur (1967) based on the work of Frankenstein and

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Garner (1967) to account for the presence of brine. The effect of temperature on the continuum behavior of ice is modelled in terms of an Arrhenius activation energy law that is considered by Mellor (1983) to be valid for temperatures below -10°C. The model is described in two papers by Ting and Shyam Sunder (1986a,b); the first focuses on the continuum behavior of sea ice while the second focuses on its yielding and fracture behavior. This paper presents the model in integrated form emphasizing its micromechanical interpretation.

CONTINUUM THEORY

Uniaxial model formulation
The nonlinear generalization of the two element Maxwell fluid model consists of an elastic spring in series with a viscous dashpot. The characterization of the two elements is discussed in what follows.

Rate-sensitive elastic spring As a consequence of its application at high homologous temperatures, the elastic modulus of ice is sensitive to even "slight" variations in rate of loading (Mellor, 1983) and cannot be taken as a constant. If the Young's modulus for ice, E, is defined to be the modulus value at very high rates of loading, then the variation of effective or apparent elastic modulus, E_{eff}, with rate may be expressed as:

$$E_{eff} = E \left[1 - r \exp(-A/E \; \dot{\epsilon}^{1/N}) \right] \tag{1}$$

where $\dot{\epsilon}$ is the strainrate, r and A are constants, and N is the power law index for ice. Equation (1) shows that the effective modulus tends to the Young's modulus as the strain-rate approaches infinity. When the strainrate tends to zero, the effective modulus tends to $(1-r)E$, and for r equal to one, the effective modulus tends to zero. If r is zero, the effective modulus is rate-insensitive and equal to the Young's modulus. A value of r less than one is necessary to model stress relaxation.

The rate-sensitive elastic spring represents recoverable strains contributed both by instantaneous elasticity and by delayed elasticity. By defining the total elastic strain to be the sum of the strains due to these components it is possible to model the rate-sensitive spring as the series combination of two springs, one with modulus equal to E, i.e., the Young's modulus, and the other with a modulus equal to E_d, the modulus of delayed elasticity. It then follows that:

$$1/E_{eff} = 1/E + 1/Ed \tag{2}$$

with

$$E_d = E [1/r \exp(A/E \, \dot{\epsilon}^{1/N}) - 1] \qquad (3)$$

Equation (3) shows that the modulus of delayed elasticity tends to infinity at infinite strainrate and to $[(1-r)/r]E$ at zero strainrate. In the latter case, the modulus tends to zero if r is one and to infinity if r is zero. Use of Eq. (2) shows that when the modulus of delayed elasticity is infinity, the effective modulus equals the Young's modulus. When the modulus of elasticity is very small with respect to the Young's modulus, the effective modulus equals the modulus of delayed elasticity. This occurs at low strainrates as r tends to one.

Nonlinear viscous dashpot The secondary creep strainrate, $\dot{\epsilon}_{sc}$, in ice is modelled in terms of the well-known Glen's power law (Glen, 1955):

$$\sigma = (A/M) \, \dot{\epsilon}_{sc}^{1/N} \qquad (4)$$

where A and N are the constants in Eq. (1), and M is a third constant. (A/M) is often taken to be a single constant. However, since A is used to describe delayed elasticity or primary creep resistance, an additional degree of freedom in the form of the constant M is used to model the secondary creep resistance. The nonlinearity is associated with the dashpot constant $\sigma/\dot{\epsilon}_{sc}$ which is a function of the secondary creep strainrate.

The description of deformations due to delayed elasticity and secondary creep proposed here can be interpreted in terms of slip within and between ice grains along the lines suggested by previous investigators. The practical value of such an interpretation is not always clear. For example, the models of Sinha (1979, 1982) and Michel (1981) consider grain boundary sliding and basal slip in the hexagonal ice crystals as factors governing the two deformations, while the model of Duval and Ashby (1985) considers slip on the basal plane plus one other nonbasal system as being responsible for the deformations. However, in all three cases the resulting "physical" model itself is a spring-dashpot idealization of the postulated slip mechanisms.

Continuous damage model In the transition from pure ductile to pure brittle behavior under compressive loading ice behaves essentially as a continuum undergoing damage. Under tensile loading the transition region is much smaller and a continuum description of damage is of limited value. Damage or micro-

cracking activity in ice leads to tertiary creep under constant stress loading and to strain-softening under constant strainrate loading. Damage is almost nonexistant at very small strain and strain-rate but increases as both strain and strainrate increase. Further, unloading a damaged material generally exhibits a reduction in the effective modulus of elasticity. Thus the phenomenon affects both the rate-sensitive spring and the non-linear viscous dashpot. The development here is based on the hypothesis that damage influences the constant A describing the creep resistance of ice, which appears in both the elements of the generalized Maxwell model. The effect of damage on the Young's modulus is neglected. Defining D as a one-dimensional damage parameter and A_D as the damaged value of the constant A, it is possible to state that:

$$A_D = (1-D)A \qquad (5)$$

where $D=0$ in the case of no damage, and the A's in Eqs. (1), (3) and (4) are replaced with A_D. In general, D varies between zero and one. For the case of total damage, i.e., $D=1$, the material looses all its ability to sustain stresses. The following mathematical form describes the dependence of D on strain and strainrate, and satisfies the physical constraints identified above.

$$D = 1 - [\exp(-c_1\varepsilon\dot{\varepsilon}) + \exp(-c_2\dot{\varepsilon})\{1-\exp(-c_1\varepsilon\dot{\varepsilon})\}] \qquad (6)$$

where c_1 and c_2 are constants. This equation shows that as strainrate approaches infinity D tends to one, i.e., the material is completely damaged. Further, as strain approaches infinity D tends to $[1-\exp(-c_2\dot{\varepsilon})]$, i.e., there is a limiting value of damage at any given strainrate.

The formulation of the damage parameter in Eq. (6) is appropriate for monotonic loading conditions. For a variable loading history the evolution of the damage parameter is assumed to follow the well-known Miner's linear damage accumulation rule. According to this rule, the incremental damage accumulation depends only on the current state of damage. Thus if the rate of loading (or strainrate) changes at some instant, Eq. (6) applies at the new rate with the level of damage previously accumulated. This requires the definition of an "equivalent" initial strain corresponding to the new strainrate.

Temperature effects   At rates of loading where no material damage is present, the effect of temperature on the stress versus secondary creep strainrate relationship is characterized by an Arrhenius activation energy law. Mellor (1983)

states that for temperatures greater than -10°C the law is not valid and that the complete empirical relation derived from experiments should be used to model the temperature dependence in such cases. Sinha (1978) has concluded that the variation of the delayed elastic or primary creep strain with temperature also follows an Arrhenius law. He found the activation energy for both the secondary creep flow and the delayed elastic deformation to be equal. The parameter A, which appears in Eqs. (1), (3) and (4), describes the creep resistance of ice and is taken to follow the Arrhenius activity energy law to model temperature effects below -10°C, i.e.,

$$A = A_0 \exp(Q/NRT) \tag{7}$$

where T is the temperature in degrees Kelvin, A_0 is a temperature independent constant, Q is the activation energy, and R is the universal gas constant. As the temperature reduces, the parameter A increases in value. In consequence, the effective elastic modulus tends to the Young's modulus and the nonlinear dashpot becomes highly viscous. Then ice displays a purely linear elastic behavior with little rate or temperature sensitivity. This model takes into account the relative temperature insensitivity of the Young's modulus of ice and of the microcracking activity in ice.

Uniaxial model parameters The uniaxial model has been successfully verified against several independent sets of experimental data for both pure and sea ice as described in Ting and Shyam Sunder (1985b, 1986a). The verification is based on the following values of the eight model parameters:

$$E = 9.5 \text{ GPa}$$
$$r = 0.98$$
$$A_0 = 0.00652 \text{ MPa s}^{1/N}$$
$$N = 3$$
$$M = 1411.2$$
$$Q = 65,000 \text{ J mol}^{-1}$$
$$c_1 = 2.28 \times 10^5 \text{ s}$$
$$c_2 = 1028 \text{ s}$$

The universal gas constant, $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$. The value of r is subject to some uncertainty since experimental data is inadequate to quantify it. Additional data may indicate that a lower value is more appropriate. All other parameters can be derived from conventional tests conducted on ice.

The models of Sinha (1978), Michel (1981), Duval and Ashby (1985), and Szyszkowski and Glockner (1985) use 7, 9, 11, and 9 parameters respectively if prediction of temperature dependence with a single parameter, for example Q, is considered.

It must be recognized that (i) the models of Sinha, and Duval
and Ashby do not capture material damage, (ii) verification of
Michel's model with experimental data appears to be limited,
and (iii) the ability of the models to describe stress
relaxation has not been investigated adequately. Furthermore,
the models have not been verified against sea ice data.

Multiaxial model formulation

Field observations have shown that sea ice, which is
predominantly columnar, has two sources of anisotropy: (a)
the c-axis is oriented perpendicular to the axis of crystal
growth, and (b) the c-axes of different crystals or grains may
show preferred azimuthal orientation in the plane on which
they lie. This anisotropy strongly influences the mechanical
behavior of first year sheet ice, while its influence on the
behavior of multi-year floes, though less well studied, may be
less. The development presented here is based on an
orthotropic generalization of the uniaxial, rate-sensitive
damage model for the continuum (flow) behavior of ice.

The strainrate vector due to the three components of creep is
related to the stress vector through an effective stress
measure for orthotropic materials with identical behavior in
compression and tension, i.e.,

$$\sigma_e^2 = 3/\beta \; [\frac{a_1}{3} (\sigma_{xx}-\sigma_{yy})^2 + \frac{a_2}{3} (\sigma_{yy}-\sigma_{zz})^2 + \frac{a_3}{3} (\sigma_{zz}-\sigma_{xx})^2$$

$$+ 2a_4 \sigma_{xy}^2 + 2a_5 \sigma_{yz}^2 + 2a_6 \sigma_{zx}^2] \qquad (8)$$

with $\beta = a_1 + a_2$. This may be expressed in compact form using
matrix notation as:

$$\sigma_e^2 = 3/\beta \; \underline{\sigma}^T \underline{G} \underline{\sigma} \qquad (9)$$

where $\underline{G}$ is a matrix containing the a_i's and defined in Ting
and Shyam Sunder (1985).

The creep strainrate vector can be related to the stress
vector with the help of a scalar potential function obeying
the associated flow rule. The desired relationship can be
written as:

$$\underline{\dot{\epsilon}}_{cr} = \lambda \; \underline{S}^* \qquad (10)$$

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where

$$\lambda = 3/\beta \ \frac{1}{\sigma_e}[\frac{\overset{\delta}{\sigma}_e}{E_d} + (M/A_D)^N \ \sigma_e^{\ N}]$$ (11)

and

$$\underline{S}^* = \underline{G}\sigma$$ (12)

Note that $\underline{S}^*$ may be thought of as a pseudo deviatoric stress vector for an anisotropic material. The evaluation of A_D and E_d requires knowledge of the effective creep strainrate vector which can be expressed in a manner similar to Eq. (9) as shown in Ting and Shyam Sunder (1985).

Given the stress vector, the pseudo deviatoric stresses may be obtained from Eq. (12). Then, applying Eqs. (9), (11) and (10) in succession leads to the creep strainrate vector.

Multiaxial model parameters Five uniaxial (compression) tests at constant strainrate may be used to derive the five ortho-tropic model parameters: a_2 to a_6. Note that (i) a_1 can be set equal to one without loss of generality, and (ii) there is experimental evidence which shows that the power-law exponent N can be considered independent of the direction of loading. The six tests (including one used to obtain the reference uni-axial parameters) can be conducted in the three directions de-fining the axes of orthotropy and along the 45° axes on the three orthogonal planes contained by the axes. Ting and Shyam Sunder (1985, 1986a) provide equations to evaluate the model parameters from the experimentally obtained ratios of stress in a given direction to that in the reference direction. In first year sheet ice, the ratio of vertical stress to horizon-tal stress in the direction of the c-axes at constant strain-rate varies from 2 to 5 (Butkovich, 1959, Peyton, 1966, Vaudrey, 1977, Sinha, 1983a, and Frederking, 1983), while the stress ratio may vary in the range 0.25-0.60 at a 45° azimu-thal angle to the c-axes when they are aligned on the horizon-tal plane and 0.50-0.95 at a 90° angle (Wang, 1979, Vittoratos, 1979, Richter-Menge, et al., 1985, Peyton, 1968).

This pressure insensitive material model for the continuum (flow) behavior of ice follows very well the experimental data of Frederking (1977) on the plane strain compressive strength of columnar grained and granular-snow ice. Verification of the model against data on sea ice, which is significantly less pressure sensitive than pure ice (see Richter-Menge, et al., 1985, for data on anisotropic first year sea ice), suggests that a pressure sensitive model may be adequate in many engineering applications.

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YIELDING AND FRACTURE THEORY

Uniaxial model formulation

The behavior of ice in uniaxial tension and compression at constant strainrates is identical up to a transition strain-rate, $\dot{\varepsilon}_{tt}$. Above this strainrate, ice fractures in tension although it continues to flow in compression. For the range of grain sizes typically encountered in sea ice, Schulson (1979, 1984a,b) and Currier and Schulson (1982) find that the tensile strength of ice is governed by the stress to nucleate cracks (not the stress to propagate cracks) and that the tensile strength actually does represent fracture, i.e., sudden failure. The ductile to brittle transition in tension tends to occur over a negligible range of strainrates unlike that in compression where strain-softening is an important behavioral phenomenon.

In compression, the strength is governed by brittle fracture only when the strainrate exceeds a limiting value, $\dot{\varepsilon}_{tc}$, which typically is greater than $\dot{\varepsilon}_{tt}$ by several (5 to 6) orders of magnitude. However, microcracks can nucleate at lower strainrates as a result of material damage. The stress at which the first "important" microcrack, i.e., one that is visible to the naked eye in pristine ice, nucleates is termed the "yield" stress in compression. This yield stress is always less than (or equal to) the maximum attainable compressive stress or "strength".

A consistent model for predicting the uniaxial fracture strength in tension and yield stress in compression are developed below.

Tensile fracture strength

The stress-strain-strainrate behavior in uniaxial tension prior to fracture is considered to be identical to that under uniaxial compression. Hawkes and Mellor (1972) justify this assumption from creep tests on ice. The tensile fracture strength versus strainrate relationship can be modeled as:

$$\frac{1}{\sigma_{tf}} = \frac{1}{\sigma_{tm}} + \frac{1}{B\dot{\varepsilon}^{1/N}} \qquad (13)$$

where σ_{tm} is the maximum and constant value of tensile strength at high strainrates and the second term models the

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increase in tensile strength as the strainrate increases from
the transition value. The value of B is chosen so that at $\dot{\varepsilon}_{tt}$
Eq. (13) predicts a value of stress, σ_{tt}, equal to that given
by the continuum theory.

The data of Ashby and Cooksley-Hallam for pure ice contained
in Palmer et al. (1982) suggests that $\dot{\varepsilon}_{tt} = 5 \times 10^{-8}$ s-1 and
$\sigma_{tt} = 0.48$ MPa. From this the material constants for the model
are estimated to be:

$$B = 176 \text{ MPa s}^{1/N}$$
$$\sigma_{tm} = 2.0 \text{ MPa}$$

Model parameters for sea ice are estimated from the above by
correcting for the presence of brine.

Compressive yield strength   Prediction of the first
"important" crack to nucleate under uniaxial compressive load-
ing is based on the hypothesis that this occurs due to the la-
teral tensile strain and strainrate resulting from the Poisson
effect of elasticity and the incompressibility condition of
creep deformation. The first crack is postulated to occur
when the lateral tensile strain equals the strain for fracture
under uniaxial tensile loading at the instantaneous strain-
rate. The strain for tensile fracture at a given strainrate
can be obtained from the tensile stress-strain-strainrate-
strength model. This results in a rate-dependent limiting
tensile strain criterion for first crack occurrence or nuclea-
tion. The adequacy of this criterion has been established
(Ting and Shyam Sunder, 1985b, 1986b) with the help of Gold's
(1972) experimental data on the time to first crack occurrence
in pure polycrystalline ice during compressive creep tests.

The compressive stress at which the first crack nucleates is
the desired yield stress, σ_{cn}. Once this stress is reached,
the material continues to sustain compressive load but loses
its ability to carry tensile loads in the transverse direction
if applied. This is a realistic modeling assumption that is
often used to describe the behavior of concrete (ASCE, 1982).
Application of the cracking criterion to constant strainrate
tests in uniaxial compression results in the following mathe-
matical model for the yield stress:

$$\frac{1}{\sigma_{cn}^2} = \frac{1}{\sigma_{cm}^2} + \frac{1}{[(A/M) \dot{\varepsilon}^{1/N}]^2} \tag{14}$$

where σ_{cm} is the maximum or constant value of yield stress in
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represents the fracture stress for strainrates greater than $\dot{\varepsilon}_{tf}$. At these strainrates both the yield and fracture stresses are almost equal and an elastic-brittle fracture model for ice is adequate. The second term models the increase in yield stress as the strainrate increases from twice the transition value in tension. The factor of two is applied since incompressibility requires the lateral tensile strainrate to be half the compressive strainrate. Below this transition strainrate the material flows in compression without nucleating cracks to cause "yielding".

Experimental data for both pure and sea ice suggests that $\dot{\varepsilon}_{tc}=10^{-2}$ s^{-1} and $\sigma_{cm}=5$ MPa. This stress value should be corrected for the presence of brine when applied to sea ice.

Multiaxial model formulation
A rate-sensitive and isotropic Drucker-Prager "failure" surface is used to describe the yield/fracture behavior of ice. The failure surface may be expressed as:

$$f(\underline{\sigma}) = p \ I_1 + \sqrt{J_2} - k \qquad (15)$$

where p and k are constants, and I_1 and J_2 are the first invariant of stress and the second invariant of deviatoric stress respectively. Thus, the model developed here considers the failure surface to be pressure sensitive although the flow behavior is considered to be pressure insensitive. The constants p and k may be derived from two uniaxial tests at constant strainrate, one in tension and the other in compression. The resulting equations are:

$$p = \frac{1}{3} \frac{(\sigma_{cn}/\sigma_{tf}) + 1}{(\sigma_{cn}/\sigma_{tf}) - 1} \qquad (16)$$

and

$$k = \sigma_{cn} \ (p - 1/\sqrt{3}) \qquad \text{or} \qquad \sigma_{tf} \ (p + 1/\sqrt{3}) \qquad (17)$$

Both p and k are functions of the effective strainrate. For strainrates less than the transition strainrate in tension, p equals zero and k is proportional to the effective stress for the continuum flow. Experimental data to develop an aniso-tropic model for the failure surface is not available at the present time.

For effective strainrates below the transition strainrate in uniaxial tension, ice flows as a continuum but does not "fail". The maximum stress state attainable at any given strainrate is given by Eq. (8) and termed the limiting flow

surface. When the effective strainrate exceeds the transition
value in uniaxial compression, it is assumed that no flow can
occur. Instantaneous elastic deformation is followed by
fracture on the Drucker-Prager surface. At intermediate
values of effective strainrate the limiting flow surface and
the failure surface intersect. Whenever one of the principal
stresses is tensile, the failure surface defines the maximum
attainable stress state prior to fracture. The direction of
cracking is assumed to be perpendicular to the largest
principal tensile stress. When the state of stress is purely
compressive, the limiting flow surface is attainable.
However, if the loading path crosses the "failure" surface, a
crack is assumed to nucleate in the direction of the smallest
compressive stress and no tensile stress can be sustained by
the material in that direction if applied at a later time.

Haynes (1973) has compared several classical failure theories
for brittle materials against his experimental data on the
tensile strength of bubbly polycrystalline ice under triaxial
stresses. None of the theories was able to adequately predict
the data. However at approximately the same strainrate used
by him, i.e., 10^{-5} s^{-1}, the model developed here indicates a
ratio of σ_{cn} to σ_{tf} equal to about 1.7. This appears to
provide the best prediction of the measured data. These
results are consistent with Schulson's (1984a, personal
communication) work on the ductile to brittle transition in
ice under compression if it is recognized that crack
nucleation (not crack propagation) governs both the tensile
fracture strength and the compressive yield stress.

CONCLUSIONS

This paper has presented a new and integrated constitutive
theory for the continuum and yielding/fracture behavior of sea
ice emphasizing its micromechanical interpretation. The model
has been extensively verified against experimental data. A
numerical solution algorithm has been developed to simulate
arbitrary loading histories using the model. This has been
implemented in a computer program based on the finite element
method of analysis. Several boundary value problems involving
ice-structure interaction have been solved using the computer
code and assuming continuum behavior of ice as described in a
companion paper at this conference (Shyam Sunder, 1986).

Current research at MIT is concerned with the incorporation of
yielding/fracture in the numerical simulation of ice-structure
interaction processes. An objective fracture energy release
rate criterion is being developed to modify the "strength"
based fracture criterion presented in this paper.

Further research, both experimental and theoretical, is neces-
sary to investigate the influence of (i) unloading/reloading
and cyclic loading, and (ii) brine volume, or more generally
porosity, on the multiaxial behavior of ice. The results of
such research can be used to improve the model proposed here.

ACKNOWLEDGEMENTS

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